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CONTIGUITY MATRIX OF SPATIAL UNITS AND ITS PROPERTIES ON EXAMPLE OF LAND DISTRICTS OF PODKARPACKIE PROVINCE

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ABSTRACT

Spatial units are an important carrier of numerical data on tested statistical characteristics of selected social-economic phenomena. They are in the relation of contiguity to one another, which can be expressed by the binary matrix of contiguity. Its 1-elements indicate occurrence of common boundary, and zeros – no boundary. In the work various divisions of spatial units have been presented as well as their properties and description of contiguity allowing determination of the contiguity matrix. It stands out with many analytical properties. Its illustration has been shown on land districts of Podkarpackie Province.

Key words: Spatial units, non-directed graphs, contiguity matrix, land districts of Podkarpackie Province

1. Introduction

Spatial units as statistical objects frequently occur in taxonomical analyses due to the tested various complex social-economic phenomena (e.g. Zeliaś 1991, 2000, Grabiński 2003). These units are carriers of multidimensional numerical data about a tested set of statistical characteristics and are evaluated in respect of their intensity. It allows creation of various types of classification of units and their illustration on statistical maps.

Spatial units are in relation of contiguity to one another, sharing the common boundary. This relation for some two units can refer to the very fact of sharing the common boundary, which is expressed binary-wise by 1 – contiguity occurs and 0 – no contiguity, or concerns occurrence of common territorial boundary expressed in its length, which is specified by real values, where contiguity occurs (e.g. km) or zero, when there is no such contiguity. Spatial contiguity can be also considered by appurtenance of common areas of forest complexes, nature reserves

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or protected landscape reserves, and the like, as well as common areas of economic activity which are expressed in surface units (e.g. ha, sq km) for each of the units.

The paper presents the characteristics of spatial units, their properties and common contiguity description giving the basis for determination of the contiguity matrix. For these matrixes various properties were given. It was presented for land districts of Podkarpackie Province.

2. Land districts of Podkarpackie Province

Statistical units used for illustration of contiguity matrix in this work are land districts of Podkarpackie Province (PDK). It is divided into 21 land districts and 4 town districts (Rzeszów, Krosno, Przemyśl and Tarnobrzeg) (fig. 1).

Figure 1. Land and town districts of Podkarpackie Province

Data on area and population number of land districts of PDK province have been given in table 1. They are arranged in ascending order according to their area. The table presents also population density per 1 sq km and the percentage participation of the area and population of land districts in their general size in the province.

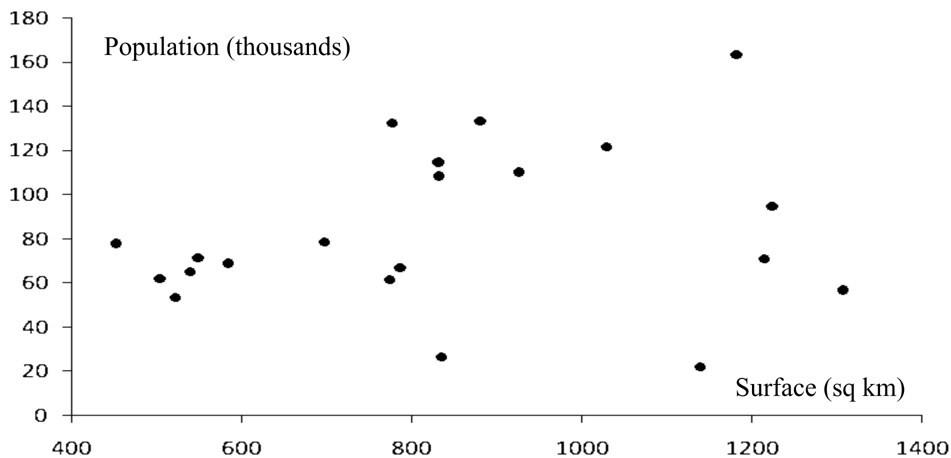
Table 1. Numerical data on districts. State on 31.12.2008

No.	Districts	Code	Area	Population	Population density	Percentage participation of	
						Area	Population
1	Łańcucki	ŁAN	452	77.9	172	2.6	4.4
2	Strzyżewski	STRZ	503	61.9	123	2.9	3.5
3	Tarnobrzeski	TAR	521	53.6	103	3.0	3.0
4	Brzozowski	BRZ	539	65.1	121	3.1	3.7
5	Ropczycko-Sędziszowski	R-S	548	71.3	130	3.1	4,0
6	Leżajski	LEŻ	584	69	118	3.3	3.9
7	Przeworski	PRZW	697	78.6	113	4.0	4.5
8	Kolbuszowski	KOL	774	61.4	79	4.4	3.5
9	Dębicki	DĘB	777	132.6	171	4.4	7.5
10	Niżański	NIŻ	786	67	85	4.5	3.8
11	Jasielski	JAS	831	114.8	138	4.7	6.5
12	Stalowoski	STAL	832	108.5	130	4.7	6.2
13	Leski	LES	835	26.5	32	4.7	1.5
14	Mielecki	MIEL	880	133.3	151	5.0	7.6
15	Krośnieński	KROŚ	926	110.3	119	5.3	6.3
16	Jarosławski	JAR	1,029	121.8	118	5.9	6.9
17	Bieszczadzki	BIESZ	1,139	22.1	19	6.5	1.3
18	Rzeszowski	RZESZ	1,182	163.4	138	6.7	9.3
19	Przemyski	PRZ	1,214	71.1	59	6.9	4.0
20	Sanocki	SAN	1,224	94.7	77	7.0	5.4
21	Lubaczowski	LUB	1,307	56.9	44	7.4	3.2
Sum			17,580	1,761.8	100	100	100

Source: The authors' elaboration on the basis of: „Powierzchnia i ludność w przekroju terytorialnym w 2008 r.", GUS, Warszawa (Area and Population in Territorial Section in 2008)

The area of land districts constitutes 98.52% of the area, and population 84.0% of number of inhabitants of PDK province. Five districts, i.e.. BIESZ, RZESZ, PRZ, SAN and LUB occupy 34.5% of the province area, but they are inhabited by only 23.2% of population (fig. 2). In respect of area the biggest one is LUB district to which belongs 7.4% of general area of the province, and the most populated is RZESZ district in which lives almost every 10th person of PDK province.

Figure 2. Correlation chart of area and population number of land districts



Source: The authors' elaboration

Districts in fig. 2 create 5 concentrations given in table 2.

Table 2. Concentrations of land districts

Concen-trations	Num-ber	Districts	Size scale	
			surface	population
1	9	ŁAN, STRZ, TAR, BRZ, R-S, LEŻ, PRZW, KOL, NIŻ	low	medium
2	6	DĘB, STAL, JAS, MIEL, KROS, JAR	medium	high
3	1	RZESZ	high	high
4	3	SAN, PRZ, LUB	high	medium
5	2	LES, BIESZ	high	low

Source: The authors' elaboration

3. Spatial units and their division

It is assumed that for a given connected area a set of n spatial units $\mathbf{J} = \{J_1, J_2, ..., J_n\}$ meeting the conditions of disjointing and covering is considered. It constitutes a complete system of units. In the formal respect, from the point of view of the theory of sets, it means fulfilment of the following conditions: covering $\bigcup_{i=1}^{n} J_i = U$ and disjointing $\bigcap_{i=1}^{n} \operatorname{int} J_i = \varnothing$, where int T means the interior of set T, and U is the divided big (basic) set.

It is assumed that for each unit there exists at least one adjacent unit, i.e. one of contiguity relations is determined:

a) with one unit

$$J_i \to \{J_{i_k}\},\ \ i_k \in I - \{i\},$$

b) with m units

$$J_i \sim \{J_{i_1}, J_{i_2}, \ldots, J_{i_m}\},\ i = 1,2,\ldots,n,$$

where $i_1, i_2, \ldots, i_{m_i}, m_i \in \{1,2,\ldots,n\}$, which means that $J_i \cap \bigcap_{p=1}^{m_i} fr\ J_{i_p} = \varnothing$, where *fr T* expresses the boundary (edge) of set T.

For the pair of units $J_i, J_k \in \mathbf{J}$ we determine their common boundary of contiguity, which we denote by the symbol $K(J_i, J_k)$. It means that for the set **J** a non-empty set of boundaries (edges) **K** occurs**.** We express the measure of common boundary by $\# K(J_i, J_k)$ (e.g. length in km, the common boundary of forest complex in km). The fact of occurrence of common boundary means that $\# K(J_i, J_k) > 0$, otherwise such a boundary does not exist and then we assume $\# K(J_i, J_k) = 0$.

For the set of edges **K** of their given measures one may carry out the following operations:

a) determining the shortest edge $K_{\min}$ and the longest edge $K_{\max}$ and the total length of all edges,
b) arranging the edges according to their lengths into a non-decreasing sequence,
c) establishing the units of the lowest and the highest number of edges,
d) selecting for each unit its longest edges, and then arranging the units in respect of these edges.

The set of spatial units **J** is divided into two disjoint subsets of boundary units $\mathbf{J}_B$ and internal units $\mathbf{J}_W$ in order that $\mathbf{J} = \mathbf{J}_B \cup \mathbf{J}_W$ and $\mathbf{J}_B \cap \mathbf{J}_W = \varnothing$. Their sizes are expressed respectively by n_B, n_W at $n = n_B + n_W$. According to the given division, subsets of boundary edges $\mathbf{K}_B$ and internal edges $\mathbf{K}_W$ are singled out.

Attachment of units to the mentioned sets is determined by assuming that all units $J_{i_1}, J_{i_2}, \ldots, J_{i_{m_i}}$ are neighbours of unit J_i. Unit J_i is called boundary unit, when $J_i \cap \bigcap_{p=1}^{m_i} fr\, J_{i_p} \neq \varnothing$, and then $J_i \in J_B$. Thus, it is obvious that when $J_i \cap \bigcap_{p=1}^{m_i} fr\, J_{i_p} = \varnothing$, then J_i is the internal unit, i.e. $J_i \in J_W$.

Spatial units of the set $\mathbf{J}_B$ have the embedding (depth) of zero degree. Higher degrees of embedding for the units $\mathbf{J}_W$ are defined by minimum number if units separating them in respect of at least one unit of the set $\mathbf{J}_B$. For example, if the units from the set $\mathbf{J}_W$ have common boundary with at least one unit from the set $\mathbf{J}_B$, then they determine the embedding of first degree.

Lets introduce the measure of distance of the units $J_i \in \mathbf{J}_B$ and $J_k \in \mathbf{J}_W$ by $D(J_i, J_k)$, which is the lowest number of boundaries (edges) of contiguity between the units J_i, J_k, which is formally expressed by

$$D(J_i, J_k) = \underset{m=1,2,\ldots,n}{\arg\min} (|\{J_1, J_2, \ldots, J_m : J_1 \sim J_i, J_m$$

$$= J_k, J_p \sim J_{p+1} \forall p = 1,2,\ldots,m-1\}|) - 1,$$

where $|T|$ means the size of set *T*.

Then all units from the set $\mathbf{J}_W$ for which $D(J_i, J_k) = 1$ have the embedding of *1st* degree, and when $D(J_i, J_k) = 2$, then they determine the embedding of *2nd* degree, etc. We denote corresponding sets of spatial units, determined in this way, by $\mathbf{J}_0 = \mathbf{J}_B, \mathbf{J}_1, \mathbf{J}_2, \ldots, \mathbf{J}_m$, where m is the maximum degree of embedding. It is obvious that $\mathbf{J}_1, \mathbf{J}_2, \ldots, \mathbf{J}_m \in \mathbf{J}_W$. For the unit $J_k \in \mathbf{J}_W$ reaching the biggest embedding it is assumed that it constitutes the centre of the set of spatial units, but there can be more central units. In the contiguity of these units there frequently is the highest number of neighbours. Similar contiguities can be considered for any unit from the set $\mathbf{J}_W$.

To any two spatial units J_i, J_k, separated by at least one unit, one or more itineraries (routes) are assigned. These itineraries constitute a set of pairs of adjacent units starting from the pair $(J_i, *)$, and ending in the pair $(*, J_k)$, where * means the unit adjacent to the one given in the pair. We denote the set of these pairs by $M(J_i, J_k)$, and its size by $\# M(J_i, J_k)$. In consequence for the earlier mentioned measure $D(J_i, J_k)$ we have

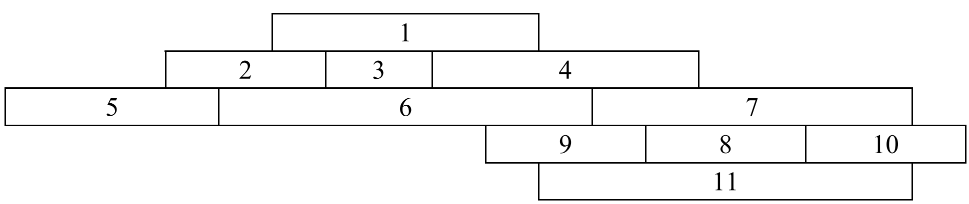
$$M(J_i, J_k) = \{(J_{i_p}, J_{p+1}) : p = 0,1,...,m-1, J_{i_0} = J_i, J_{i_m} = J_k\}.$$

Among pairs of units there are itineraries containing the lowest number of boundaries (edges) which should be overcome, coming through other units, and we will deal with such ones later on. The longest itineraries are always connected with the units of the set $\mathbf{J}_B$ and we denote them by $M^*(J_i, J_k)$. For $M(J_i, J_k)$ the inequality $M^*(J_i, J_k) \geq M(J_i, J_k)$ is fulfilled. The itineraries $M^*(J_i, J_k)$ constitute the diameter of the set of spatial units. There can be a few of such diameters. The units coming into the composition of a given itinerary without boundary units constitute transitive units. Itineraries containing at least 3 units, starting and ending in the same units, are defined as cycles, i.e. when $i = k$ and $| M(J_i, J_k) | \geq 3$.

A number of adjacent units belongs to each unit. It means the assignment to the unit J_i of n_i other units respectively, at $n_i \geq 1$. Such an assignment creates a list of contiguity. The contiguity of units is expressed with the use of a non-directed graph (e.g. Wilson 2004). It is determined by the pair $\mathbf{G} = (\mathbf{J}, \mathbf{K})$ of the set of units (knots) $\mathbf{J}$ and edges (boundaries) $\mathbf{K}$.

We will consider the presented notions on the set of 11 spatial units arranged in the area shown in fig. 2.

Figure 2. Connected area with singled out spatial units



Source: the authors' elaboration

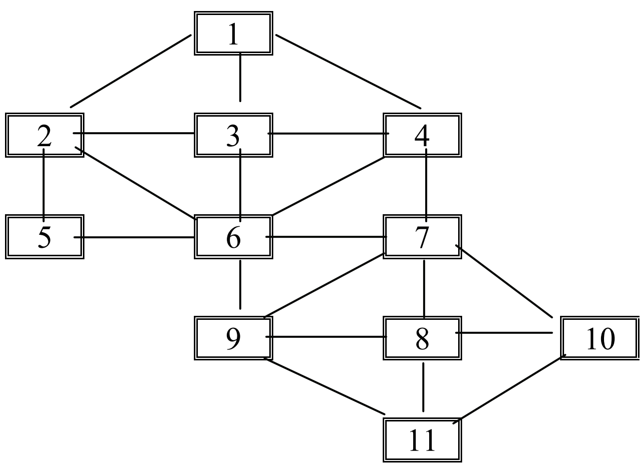
For the units presented in fig. 2, we have respectively:

- $n = 11$, $\mathbf{J} = \{J_1, J_2, ..., J_{11}\}$,
- $\mathbf{J}_B = \{J_1, J_2, J_4, J_5, J_6, J_7, J_9, J_{10}, J_{11}\}$, $q_B = 9$,
- $\mathbf{J}_W = \{J_3, J_8\}$, $q_W = 2$,
- units J_3, J_8 are the units of embedding of 1^{st} degree,

- contiguities for units J_3 and J_8 constitute sets $\{J_1, J_4, J_6, J_2\}$ and $\{J_7, J_{10}, J_{11}, J_9\}$, whose elements are noted in clock-wise order,
- $M(J_1, J_6) = \{(J_1, J_2), (J_2, J_6)\}$, besides there exist other itineraries, then $\{(J_1, J_3), (J_3, J_6)\}$ or $\{(J_1, J_4), (J_4, J_6)\}$,
- $M(J_1, J_{11})$ maximum itinerary composed of two pairs of units, e.g. $\{(1,4), (4,6), (6,9), (9,11)\}$, $\{(1,4), (4,7), (7,9), (9,11)\}$, $\{(1,4), (4,7), (7,8), (8,11)\}$, etc.
- model non-directed graph of contiguity of units has been presented in fig. 3.

For the non-directed graph the pairs (*u, v*), (*v, u*) are equivalent, therefore at specification of the edge of set **K** only pairs (*i, k*) for $1 \le i < k \le n$ are given. For the mentioned graph the set of edges **K** = {(1,2), (1,3), (1,4), (2,3), (2,5), (2,6), (3,4), (3,6), (4,6), (4,7), (5,7), (6,7), (6,9), (7,8), (7,9), (7,10), (8,9), (8, 10), (8,11), (9,11), (10,11)} contains their 21 pairs.

Figure 3. Non-directed graph of contiguity of spatial units from fig. 2



Source: The authors' elaboration

3. Determining contiguity matrix and its properties

The matrix of contiguity **S** of the non-directed graph **G** is a binary square matrix. Its rows and columns correspond to the numbers of spatial units. 1-element occurs for the pair of units sharing common edge (boundary), otherwise 0-element appears. Such an element occurs also for the pair of identical units. Thus, the elements of matrix $\mathbf{S} = (s_{ij}), i, j = 1,2,...,n$ are determined by

$$s_{ij} = s_{ji} = \begin{cases} 1, & (i,j) \in \mathbf{K} \\ 0, & (i,j) \notin \mathbf{K} \end{cases}.$$

For the units in fig. 2 the contiguity matrix takes the form

$$\mathbf{S} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

The presented matrix contains *42* elements of the value *1*, i.e. it is the doubled product of the number of edges (size of the set **K**). For the matrix of contiguity **S** the following properties occur:

a) it is a symmetrical matrix,
b) diagonal elements equal zero,
c) the number of 1-elements equals $q = 2 \cdot \# \mathbf{K}$, where #**K** expresses the size **K**,
d) $\mathbf{S1} = \mathbf{m} -$ the product of matrix **S** and unit vector **1** gives vector **m** whose components express the number of neighbours of each spatial unit,
e) the equation $\mathbf{1'S1} = \mathbf{1'm} = q$ occurs,
f) determinant $\det(\mathbf{S})$ can be negative, zero or positive,
g) the value of the determinant of matrix **S** is invariant at change of numeration of spatial units,
h) **SS** =**T** = (t_{jk}), for *j, k = 1, 2, ..., q* – the product of contiguity matrix per se, being the matrix of elements t_{jk} corresponding to the number of neighbours of *j*-th spatial unit, when *j* = *k*, the number of common neighbours for two different spatial units being in direct contiguity or distant from each other by one unit (transitive by one), when $j \neq k$, but $t_{jk} = 0$ for pairs of units distant from each other by two or more number of units,
i) $diag(\mathbf{T}) = diag(\mathbf{m}) -$ diagonal elements of matrix **T** correspond to the components of vector of sizes of neighbours **m**,
j) $\mathbf{T} = diag(\mathbf{m}) + \mathbf{B} -$ matrix **T** is expressed by the diagonal matrix of components of vector **m** and the matrix **B** of the number of common direct and transitive neighbours,

k) $tr(\mathbf{T}) = q$ – the trace of the matrix **T** equals the number of 1-elements of the matrix **S**,

l) the equation $\det(\mathbf{T}) = \{\det(\mathbf{S})\}^2$ occurs,

m) all characteristic values of matrix **S** are real,

n) for matrix **S** one can create its submatrix containing a subset of *k* spatial units $\{i_1, i_2, ..., i_k\} \in \{1, 2, ..., n\}$ meeting the assumptions:

(i) $i_1 < i_2 < ... < i_k$ and $i_j - i_{j-1} = 1$.

(ii) $\# K(J_{i_{j-1}}, J_{i_j}) > 0$,

the conditions (i) and (ii) guarantee that by removing numbers $i_1, i_2, ..., i_k$ of rows and columns from the initial matrix S, a separated from the initial domain, still compact subdomain of spatial units will arise,

o) $\mathbf{S} = \mathbf{S}_0 + \mathbf{S}_1 + ... + \mathbf{S}_k$ – it is possible to present contiguity matrix in the form of the matrix of the sum of *k* contiguity matrixes for the created *k* subsequences of spatial units, where all matrixes are of dimensions $n \times n$,

p) $\mathbf{S} = \mathbf{S}_0 + \mathbf{S}_1 + ... + \mathbf{S}_m$ – contiguity matrix is expressed by the sum of contiguity matrixes of units of embedding of zero, first, …, m-th degrees, but all matrixes on the right side are created for the full set of *n* spatial units,

q) principal minors $\mathbf{M}_1, \mathbf{M}_2, .\mathbf{M}_3, ..., \mathbf{M}_n$ of matrix **S** are contiguity matrixes of the set of successively first unit, two first units, three first units, etc. and of all units since in the last case $\mathbf{M}_n = \mathbf{S}$.

Upper-triangular matrix **B** for the considered example takes the form:

	1	2	3	4	5	6	7	8	9	10	11
1	0	1 (3)	2 (2, 4)	1 (2)	1 *(2)	3 *(2, 3, 4)	1 *(4)	0	0	0	0
2		0	2 (1, 6)	3 *(1, 3, 6)	1 (6)	2 (3, 5)	1 *(6)	0	1 *(6)	0	0
3			0	2 (1, 6)	2 *(2, 6)	2 (2, 4)	2 *(4, 6)	0	1 *(6)	0	0
4				0	1 *(6)	2 (3, 7)	1 (6)	1 *(7)	2 *(6, 7)	1 *(7)	0
5					0	1 (2)	1 *(6)	0	1 *(6)	0	0
6						0	2 (4, 9)	2 *(7, 9)	1 (7)	1 *(7)	1 *(9)
7							0	2 (9, 10)	2 (6, 8)	1 (8)	3 *(8, 9, 10)
8								0	2 (7, 11)	2 (7, 11)	2 (9, 10)
9									0	3 *(7, 8, 11)	1 (10)
10										0	1 (8)
11											0

where in the notation two variants for the pair of spatial units have been applied: number (numbers of adjacent units) and number *(numbers of transitive adjacent units):

To spatial units there is attributed the variant of density of contiguity $\mathbf{g} = \frac{1}{n-1}\mathbf{m}$ and their average density of contiguity of all units $\bar{g} = \frac{\mathbf{1'm}}{n(n-1)}$.

4. Contiguity matrix for land districts of Podkarpackie Province

Fig. 1 presents the map of $n = 21$ land districts of PDK province. Its simplified version is presented in fig. 4.

Figure 4. Diagram of localizations of land districts of PDK province

<table>
<tr><td rowspan="2">MIEL</td><td>TAR</td><td>STAL</td><td>NIŻ</td><td rowspan="3">PRZW</td><td>LUB</td></tr>
<tr><td>KOL</td><td rowspan="2">RZESZ</td><td>LEZ</td><td rowspan="2">JAR</td></tr>
<tr><td>DĘB</td><td>R-S</td><td>ŁAŃ</td></tr>
<tr><td rowspan="2">JAS</td><td>STRZ</td><td>BRZ</td><td colspan="3">PRZ</td></tr>
<tr><td>KRO</td><td>SAN</td><td>LES</td><td>BIESZ</td><td></td></tr>
</table>

Source: The authors' elaboration.

Table 3 gives the list of contiguity according to the arrangement of land districts in fig. 1. It also includes the vector of contiguity density (Density) and the degree of embedding (Embedding), but the districts have been listed in the order of degree of embedding and number of contiguity.

Table 3. List of contiguity of districts together with density and degree of embedding

Codes	Number	Density	Embedding	Districts of contiguity								
PRZW	6	0.30	0	LUB	JAR	PRZ	RZESZ	ŁAŃ	LEŻ			
PRZ	6	0.30	0	PRZW	JAR	BIESZ	SAN	BRZ	RZESZ			
SAN	5	0.25	0	BRZ	PRZ	BIESZ	LES	KROŚ				
DĘB	4	0.20	0	MIEL	R-S	STRZ	JAS					
KROŚ	4	0.20	0	STRZ	BRZ	SAN	JAS					
LEŻ	4	0.20	0	NIŻ	PRZW	ŁAN	RZESZ					
MIEL	4	0.20	0	TAR	KOL	R-S	DĘB					
NIŻ	4	0.20	0	STAL	LEŻ	RZESZ	KOL					
BIESZ	3	0.15	0	PRZ	LES	SAN						
JAR	3	0.15	0	LUB	PRZ	PRZW						
JAS	3	0.15	0	DĘB	STRZ	KROŚ						
STAL	3	0.15	0	NIŻ	KOL	TAR						
TAR	3	0.15	0	STAL	KOL	MIEL						
LES	2	0.10	0	BIESZ	SAN							
LUB	2	0.10	0	JAR	PRZW							
RZESZ	9	0.45	1	KOL	NIŻ	LEŻ	ŁAN	PRZW	PRZ	BRZ	STRZ	R-S
KOL	6	0.30	1	TAR	STAL	NIŻ	RZESZ	R-S	MIEL			
STRZ	6	0.30	1	R-S	RZESZ	BRZ	KROŚ	JAS	DĘB			
BRZ	5	0.25	1	RZESZ	PRZ	SAN	KROŚ	STRZ				
R-S	5	0.25	1	KOL	RZESZ	STRZ	DĘB	MIEL				
ŁAN	3	0.15	1	LEŻ	PRZW	RZESZ						

Source: The authors' elaboration.

The districts show various degree of embedding. Zero embedding is shown by boundary districts whose number is *15,* and the remaining *6* districts have embeddings of *1* degree, which was marked in table 2 with digital symbols *0* and *1*. The neighbours of each of the districts were given in clock-wise direction, always starting from the northern side.

The division of spatial units, presented in table 3, into boundary and internal ones, can be supplemented with various numerical characteristics, given as an example in table 4.

Table 4. Selected numerical characteristics for boundary and internal districts

Specification	Boundary districts		Internal districts		Total	Quotient
	Value	%	Value	%		
Number of districts	15	71.43	6	28.57	21	0.40
Number of neighbours	56	62.22	34	37.78	90	0.61
Average number of neighbours	3.73	–	5.70	–	4.29	–
Area [sq km]	13,582	77,26	3,998	22.74	17,580	0.29
Population [thousand]	1,261	71,56	501	28.44	1,761.8	0.40

Source: The authors' elaboration

The last column of table 4 shows the quotients of values of characteristics of internal and boundary units. For *15* boundary districts *56* boundaries of contiguity have been determined, which gives the average of *3.73* neighbours per district, and 6 internal districts have *34* boundaries of contiguity with the average of *5.7* neighbours per one district. Boundary and internal districts constitute respectively *71.43*% and *28.57*% of their general number, and the percentages of the number of neighbours for these groups are 62.22% and 37.78% respectively.

At a more general approach of the conducted comparative analysis of the given system of statistical characteristics for boundary and internal spatial units, one can apply statistical tests of significance (e.g. Student's t-test) for individual characteristics or Hotelling's test for the set of characteristics (e.g. Morrison 1990).

The list of contiguity given in table 3 allows creating the square contiguity matrix **S** of dimensions 21×21, or the triangular matrix (most often lower-triangular, without marking zero diagonal elements). Table 5 presents the contiguity matrix of districts of PDK province.

Table 5. Contiguity matrix S for districts of PDK province

No.	Dist.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	Sum
1	TAR	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
2	STAL	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
3	NIŻ	0	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	4
4	MIEL	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	4
5	KOL	1	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	6
6	LEŻ	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	4
7	LUB	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	2
8	DĘB	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	4
9	R-S	0	0	0	1	1	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	5
10	RZESZ	0	0	1	0	1	1	0	0	1	0	1	1	0	1	1	1	0	0	0	0	0	9
11	ŁAN	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	3
12	PRZW	0	0	0	0	0	1	1	0	0	1	1	0	1	0	0	1	0	0	0	0	0	6
13	JAR	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	3
14	STRZ	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1	0	0	0	6
15	BRZ	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	1	1	0	0	5
16	PRZ	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0	0	1	0	1	6
17	JAS	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	3
18	KROŚ	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1	0	0	4
19	SAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1	1	5
20	LES	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	2
21	BIESZ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	3
Sum		3	3	4	4	6	4	2	4	5	9	3	6	3	6	5	6	3	4	5	2	3	90

Source: The authors' elaboration.

The given contiguity matrix is characterised by the following properties:

- there were 90 contiguity boundaries in total,
- numbers of neighbours of individual districts are contained in the last row and column in table 5,
- the highest number of neighbours has RZESZ (9 neighbours), and the lowest LUB (2),
- the distribution of number of neighbours for districts is given in the statement

Number of neighbours	2	3	4	5	6	9	Sum
Number of districts	2	6	5	3	4	1	21
Percentage part	9.52	28.57	23.81	14.29	19.05	4.76	100.00

- probability distribution of number of neighbours

Number of neighbours	2	3	4	5	6	9
Probabilities	0.0952	0.2857	0.2381	0.1429	0.1905	0.0476

and selected numerical characteristics of the distribution

Expected value	Standard deviation	Asymmetry
4.286	1.637	0.968

- the value of the determinant equals –168, i.e. matrix **S** is negative definite, and the sizes of successive determinants of principal minors are given in the statement:

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
-1	0	1	-2	0	0	0	0	0	0	-3	16	-16	-20	21	80	-116	80	116	-168

The product of the contiguity matrix per se leads to the matrix **T** in which the order of districts is the same as in matrix **S** given in table 5:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	3	1	2	1	2	0	0	1	2	1	0	0	0	0	0	0	0	0	0	0	0
2	1	3	1	2	2	1	0	0	1	2	0	0	0	0	0	0	0	0	0	0	0
3	2	1	4	1	2	1	0	0	2	2	2	2	0	1	1	1	0	0	0	0	0
4	1	2	1	4	2	0	0	1	2	2	0	0	0	2	0	0	1	0	0	0	0
5	2	2	2	2	6	2	0	2	2	2	1	1	0	2	1	1	0	0	0	0	0
6	0	1	1	0	2	4	1	0	1	3	2	2	1	1	1	2	0	0	0	0	0
7	0	0	0	0	0	1	2	0	0	1	1	1	1	0	0	2	0	0	0	0	0
8	1	0	0	1	2	0	0	4	2	2	0	0	0	2	1	0	1	2	0	0	0
9	2	1	2	2	2	1	0	2	5	2	1	1	0	2	2	1	2	1	0	0	0
10	1	2	2	2	2	3	1	2	2	9	2	3	2	2	2	2	1	2	2	0	1
11	0	0	2	0	1	2	1	0	1	2	3	2	1	1	1	2	0	0	0	0	0
12	0	0	2	0	1	2	1	0	1	3	2	6	2	1	2	2	0	0	1	0	1
13	0	0	0	0	0	1	1	0	0	2	1	2	3	0	1	1	0	0	1	0	1
14	0	0	1	2	2	1	0	2	2	2	1	1	0	6	2	2	2	2	2	0	0
15	0	0	1	0	1	1	0	1	2	2	1	2	1	2	5	2	2	2	2	1	2
16	0	0	1	0	1	2	2	0	1	2	2	2	1	2	2	6	0	2	2	2	1
17	0	0	0	1	0	0	0	1	2	1	0	0	0	2	2	0	3	1	1	0	0
18	0	0	0	0	0	0	0	2	1	2	0	0	0	2	2	2	1	4	1	1	1
19	0	0	0	0	0	0	0	0	0	2	0	1	1	2	2	2	1	1	5	1	2
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	1	1	2	1
21	0	0	0	0	0	0	0	0	0	1	0	1	1	0	2	1	0	1	2	1	3

On the main diagonal of matrix **T** the number of neighbours of individual districts can be read, and the elements in the intersection of a given row (*i*-th district) and column (*j*-th district) indicate their joint number of neighbours. In the case given they assume only the values 0, 1, 2 and 3. For example, for NIŻ district (3rd column) joint neighbours are presented in the statement:

J	District	Number	Joint neighbours
1	TAR	2	STAL, KOL
2	STAL	1	KOL
4	MIEL	1	KOL
5	KOL	2	STAL, RZESZ
6	LEŻ	1	RZESZ
9	R-S	2	KOL, RZESZ
10	RZESZ	2	KOL, LEZ
11	ŁAN	2	RZESZ, LEZ
12	PRZW	2	LEZ, RZESZ
14	STRZ	1	RZESZ
15	BRZ	1	RZESZ
16	PRZ	1	RZESZ

RZESZ district, which sometimes functions as a transitory (transitive) district, appeared most often in the role of a neighbour. The number of joint neighbours of individual districts is contained in the statement

Districts	TAR	STAL	NIŻ	MIEL	KOL	LEŻ	LUB	DĘB	R-S	RZESZ	ŁAN
Number	10	10	18	14	22	18	7	14	24	36	16
Districts	PRZW	JAR	STRZ	BRZ	PRZ	JAS	KROŚ	SAN	LES	BIESZ	
Number	21	11	24	25	25	11	15	15	6	10	

RZESZ districts appears in the role of joint neighbour as many as 36 times, and STRZ and BRZ districts by 25 and 24 times respectively. Hence, the conclusion is that the district with the highest number of neighbours (RZESZ – 9) appears most frequently as a joint neighbour of other districts (RZESZ – 36).

For the given order of spatial units (fig. 4) the elements in matrix **T** are grouped mainly along the diagonal.

5. Summary

Contiguity matrix appears in the theory of graphs (see: e.g. Kulikowski 1986, Wilson 2004, http://pl.wikipedia.org/wiki/Macierz sąsiedztwa 10.07.2007). Its function is, among other things, to describe graphs, and its elements express then a number of edges between vertices.

In the work the notion of contiguity matrix has been used to determine mutual positions of spatial units. Its structure is simple, and the only inconvenience is its considerable dimension when it refers to a numerous set of units. The considered matrix is characterised by many properties, of which particularly interesting is the product of this matrix per se.

Contiguity matrixes allow broadening the interpretation of researched social-economic phenomena. Its elements can be used to determine connected domains, constituting some subsets of spatial units, when their considered set is grouped into disjoint classes, which is met in taxonomical methods.

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